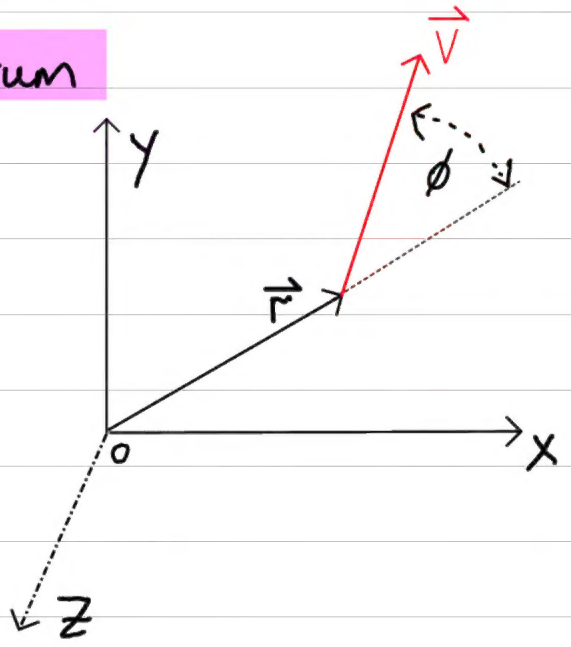
Meriam 3/10 : Angular Momentum

Consider a Particle P moving in the x-y plane



- position $\vec{r}$
- velocity $\vec{V}$
- Angle ϕ between $\vec{r}$ and $\vec{V}$

Definition :

Angular momentum of particle P about O:

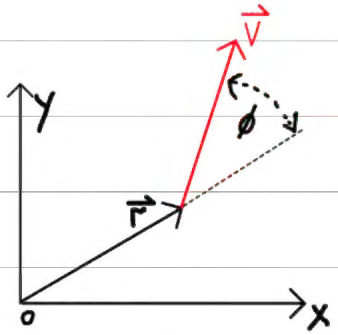
$$\vec{H}_o = \vec{r} \times m\vec{V}$$

Note:

- Angular momentum is defined wrt reference point
- H_o is a vector. Get direction with the right hand rule.
- For a 2D problem in the x-y plane,

$$|\vec{H}_o| = m\,|\vec{r}|\,|\vec{V}|\,|\sin\phi|$$

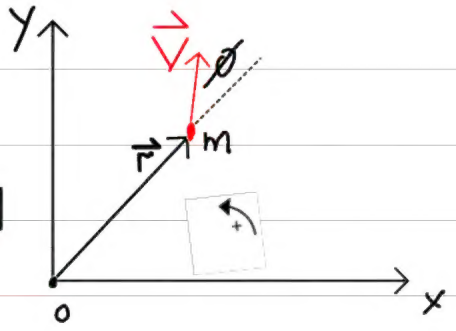
→ $\vec{H}_o = 0$ if $\phi = 0$ or 180°



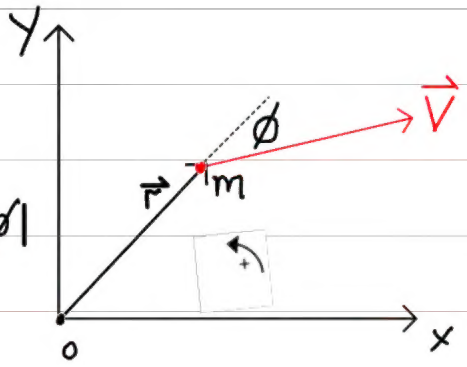
- For 2D motion in the x-y plane, $\vec{H}_o$ is oriented $\pm\hat{k}$

→ Instead of specifying the direction of $\vec{H}_o$ as a vector, for 2D x-y motion we can just specify a $\pm$ sign

$$\vec{H}_o = +\, m\, |\vec{r}|\, |\vec{v}|\, |\sin\phi|$$

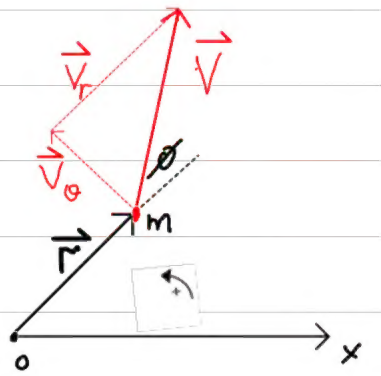


$$\vec{H}_o = -\, m\, |\vec{r}|\, |\vec{v}|\, |\sin\phi|$$



→ Better physical insight into angular momentum

- Note that for motion in the x-y plane,



$$V_r = |\vec{V}| \cos\phi$$
$$V_\theta = |\vec{V}| \sin\phi$$

$$\rightarrow \quad |\vec{H}_o| = m\,|\vec{r}|\,|\vec{V}|\,|\sin\phi| = m r |V_\theta|$$

$$\frac{d\vec{H}_o}{dt} = \frac{d}{dt}\left(\vec{r} \times m\vec{V}\right)$$

$$= \dot{\vec{r}} \times m\vec{V} + \vec{r} \times m\dot{\vec{V}}$$

($\dot{\vec{r}} = \vec{V}$, $\dot{\vec{V}} = \vec{a}$)

$$= 0 + \vec{r} \times \vec{F}$$

$$\boxed{\frac{d\vec{H}_o}{dt} = \vec{M}_o}$$

integrate:

$$\boxed{(\vec{H}_o)_2 = (\vec{H}_o)_1 + \int_{t_1}^{t} \vec{M}_o \, dt'}$$

rotation

compare with

$$\boxed{\vec{G}_2 = \vec{G}_1 + \int_{t_1}^{t_2} \vec{F} \, dt}$$

translation

A small block of mass m = 2 kg is swinging from a rope of length 3.5 meters, which is tied to the ceiling at point "O" as shown. At the instant shown, the block has a speed of 4 m/s (moving counterclockwise around the origin), and the angle θ between the x-axis and rope is 200 degrees.

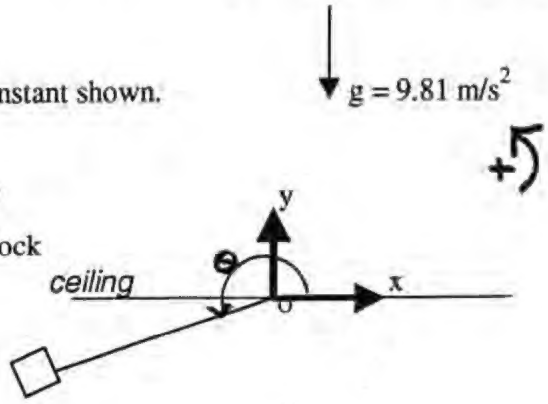
(a) What is $\dot{\theta}$ at the instant shown?

(b) Find the angular momentum of the block about point "O" at the instant shown.

(c) Find the tension in the rope at the instant shown.

(d) What will be the speed of the block when θ reaches 270 degrees?

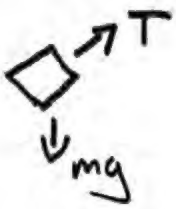
(e) **Bonus question:** At what rate is the angular momentum of the block about the point "O" changing when θ reaches 300 degrees?



(a) $\dot{\theta} = \frac{V_\theta}{r} = \frac{4}{3.5} = 1.14\ s^{-1}$ ↺

(b) $\vec{H}_o = r m V_\theta = (3.5)(2)(4) = 28\ kg\cdot m^2/s$ ↺

(c)



$\Sigma F_n = m a_n$

$T - mg\sin 20 = m V_\theta^2 / r$

$T - (2)(9.81)\sin 20° = (2)(4)^2/3.5$

$T - 6.71 = 9.14$

$T = 15.85\ N$

(d) $T_2 = T_1 - mg\Delta h$

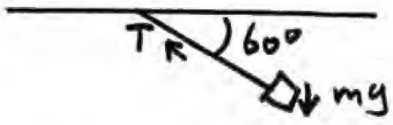
$\frac{1}{2} m V_2^2 = \frac{1}{2} m V_1^2 - (mg)(-[3.5][1 - \sin 20])$

$(\frac{1}{2})(2) V_2^2 = (\frac{1}{2})(2)(4)^2 + (2)(9.81)([3.5][0.658])$

$V_2^2 = 16 + 45.2 = 61.2$

$V_2 = 7.8\ m/s$

(e)



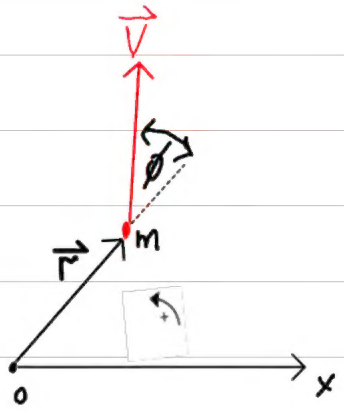
$\dot{\vec{H}}_O = \Sigma \vec{M}_O = mgr\cos 60$ ↻

$= (2)(9.81)(3.5)(0.5) = 34.33\ kg\cdot m^2/s^2$ ↻

Or $-34.33\ kg\cdot m^2/s^2$

Summary

- **Linear Momentum:** $\vec{G} = m\vec{v}$
- **Angular Momentum:** $\vec{H}_o = \vec{r} \times m\vec{v}$



- $$\frac{d(m\vec{v})}{dt} = \dot{\vec{G}} = \sum \vec{F}_i \quad \rightarrow \quad \vec{G}_2 = \vec{G}_1 + \int_{t_1}^{t_2} \sum \vec{F}_i \, dt$$

$$\dot{\vec{H}}_o = \sum \vec{M}_o \quad \rightarrow \quad (\vec{H}_o)_2 = (\vec{H}_o)_1 + \int_{t_1}^{t_2} \sum \vec{M}_o \, dt$$

For Next Lecture

- Read 4/1 – 4/5 : Systems of Particles